Philadelphia Area Number Theory Seminar

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Local and adelic Hecke algebra isomorphisms

Abstract: We will discus several aspects of the following question, which is anabelian in nature: to what extent does the representation theory of the Hecke algebra for a linear algebraic group G over a field K determine K?

First, let K and L be number fields and let G be a linear algebraic group over $\mathbb{Q}$. Suppose that there is a topological group isomorphism of points on G over the adèles of K and L, respectively. We establish conditions on the group G, related to the structure of its Borel groups, under which K and L have isomorphic adèle rings. We will discuss the concepts of local isomorphism and arithmetic equivalence of two number fields, to interpret how this result "determines" K and L.

As a corollary, we show that when K and L are Galois over $\mathbb{Q}$, an isomorphism of Hecke algebras for $\mathrm{GL}(n)$, which is an isometry in the L^1-norm, implies that K and L are isomorphic as fields. This can be viewed as an analogue in the theory of automorphic representations of the theorem of Neukirch that the absolute Galois group of a number field determines the field if it is Galois over $\mathbb{Q}$.

Secondly, let K and L be non-archimedean local fields of characteristic zero. Analogous results to the number fields case still hold. However, we show that the Hecke algebra for $\mathrm{GL}(2)$ for any local field is Morita equivalent to the same complex algebra, determined by the Bernstein decomposition.

The results in this talk were obtained during my PhD at Utrecht University, the Netherlands. Some results are joint work with Gunther Cornelissen.

Wednesday, October 26, 2016, 3:10–4:30PM

Bryn Mawr College, Park Science Center **328**

Tea and refreshments at 2:50PM in Park 355