BI-CO MATHEMATICS COLLOQUIUM

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"Using the heat equation to shorten curves (and other Lagrangian submanifolds)"

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Talk at 4:00 – E309
Tea at 3:00 – KINSC Math Lounge, H208

Abstract:
The heat equation tends to average objects into more homogeneous, symmetric, or canonical forms. In the past two or three decades, heat-type equations have led to remarkable progress in low-dimensional topology and geometry, of which the Hamilton-Perelman solution of the Poincar\'{e} Conjecture is the most famous example.

One early success in the application of the heat equation to geometry is the theorem of Hamilton-Gage and Grayson that ``The Heat Equation Shrinks Embedded Plane Curves to Round Points." In this talk, we'll make precise what it means for a curve in the plane to diffuse as though it were heat, and see how the heat equation's averaging tendency pushes any simple, closed curve closer to the canonical simple, closed curve: the circle. The techniques required to prove this remarkable theorem turn out to be quite elementary. In particular, the area enclosed by the curve is of fundamental importance to understanding how exactly the heat equation shrinks the curve to a ``round point".

Time permitting, we'll turn briefly to a higher-dimensional version of the same problem: the Lagrangian mean curvature flow. We'll see that ``enclosed area" again has much to say about what happens as the heat equation collapses a Lagrangian submanifold.

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