BI-CO MATHEMATICS COLLOQUIUM

John Boller
University of Chicago

"A little bit of p-adic analysis"

Monday, November 4, 2013

Talk at 4:00 – Park 338
Tea at 3:30 – Park 355, Math Lounge

Abstract:

What are the real numbers? In real analysis, we learn that they are the unique ordered field in which the least upper bound property holds. But where do they come from? One means of constructing the real numbers **R** is a process known as (analytic) completion, where we take equivalence classes of Cauchy sequences of rational numbers and endow them with the arithmetic of a field and an ordering.

This process of completion requires us to have an absolute value on the rational numbers **Q** in the first place. But it turns out that there are infinitely many different absolute values on **Q**! In addition to the usual one, there is one for every prime p. When we complete **Q** with respect to the absolute value associated with the prime p, we get the p-adic field $\mathbf{Q}_p$. These p-adic fields are quite different in character from the real numbers, but they are still complete, so we can ask all of the usual questions from Calculus and analysis about differentiation, integration, power series, Fourier series, and more. Some of the results will surprise!

BRYN MAWR COLLEGE