BI-CO MATHEMATICS COLLOQUIUM

Eugenio Calabi
Professor Emeritus of Mathematics
University of Pennsylvania

"The Affine Geometry of Convex Bodies"

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Talk at 4:00 – Park 328
Tea at 3:30 – Park 355, Math Lounge

Abstract:

Affine geometry in the plane finds an application in problems of pattern recognition for images under large distance perspective. Thus, for instance, straight calligraphic script can be transformed into slanted script, and squares become indistinguishable from general parallelograms. We limit our attention to the set of affine shapes of all convex bodies in the affine plane, that is to say, of regions bounded by simple, closed, convex curves (ovaloids).

There is a non-symmetric metric, a non-negative number representing a "distance" from the affine shape of a convex body A to that of B, given by the logarithm of the ratio between areas of B' and that of A,

$$d(A, B) = \log(\text{area}(B')/\text{area}(A)),$$

where B' is a tightest fitting convex body affinely equivalent to B and containing A. A symmetrized distance function is represented by $D(A, B) = d(A, B) + d(B, A)$; for instance, if A and B are respectively a square and a circle, we have:

$$d(A, B) = \log(\pi/2)\ ,\ d(B, A) = \log(4/\pi)\ ,\ \text{and}\quad D(A, B) = \log 2.$$

The main result is a description and construction of all the convex bodies A in the plane that achieve the maximum possible value of each of the three distance functions, d(A, B), d(B, A), and D(A, B), when B is a triangle: the problems in the first two cases, were solved almost 100 years ago, respectively by W. Gross and by W. Blaschke. The third case, unpublished, leads to a more complicated dynamical system, with many different solutions.

References

[1] Blaschke, Wilhelm, Berichte.sächs. Akad. Leipzig, 69 (1917), pp. 431–435.
[2] Groß (Gross), W., Eine Extremeigenschaft des Parallelogramms ("An Extremal Property of the Parallelogram"), Ber. d. sächs. Akad., Leipzig, 70 (1918), p. 40

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