BI-CO MATHEMATICS COLLOQUIUM

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"Algebraic K-theory and Its Connections to Other Fields"

Monday, November 25, 2019

Talk at 4:00 – Park 338

Tea at 3:30 – Park 361, Math Lounge

Abstract:

Studying geometric objects by associating algebraic invariants to them is a powerful idea which has influenced many areas of mathematics. The first such example is the Euler characteristic, a number associated to a polyhedron, which allowed Euler to prove that there are only 5 platonic solids. Emmy Noether shifted the focus of algebraic topology in the late 1920s from numerical invariants, to the formal algebraic objects underlying them, such as groups. More recently, the focus of algebraic topology has ascended further from groups to moduli spaces, infinite-dimensional spaces whose set of connected components recovers the underlying numerical invariant. This perspective, encapsulated in homotopy theory, replaces the quick count of the Euler characteristic ("how many vertices are there?") with spaces, deformable and manipulable through homotopy, such that one can collapse back to the enumerative world through the counting of connected components. Algebraic K-theory is a generalization of the Euler characteristic in this sense.

Algebraic K-groups are deep invariants of rings, which hide beautiful patterns and connections to problems in number theory. Lower K-groups have explicit algebraic descriptions, but higher algebraic K-groups require sophisticated topological and categorical machinery to define, and their introduction by Quillen was the culminating point of a long search for a definition that would meaningfully generalize the existing definitions of lower K-groups. Waldhausen's extension of algebraic K-theory to spaces is central to the classification of diffeomorphism groups of manifolds. In this talk, I will give a flavor of algebraic K-theory and its history, touching on current research.

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