Digitally Restricted Ostrowski Expansions

Ellie Lew ’25

The standard middle-thirds Cantor set is the set of $x \in [0,1]$ having base-3 expansions which do not contain the digit 1. The Hausdorff dimension of the Cantor Set is log 2/log 3. In fact, any Cantor-like set with digital restrictions in base b > 1 has Hausdorff dimension log a/log b, where a is the number of allowed digits. We will discuss recent work where we determine the Hausdorff dimensions of analogously defined sets. Our sets consist of real numbers whose “Ostrowski digits" have been restricted.

Fix an irrational $\beta \in (0,1)$ with continued fraction $\beta = [0;b_1,b_2,\ldots]$, and denote $D_n = q_n\beta - p_n$ where p_n/q_n are the convergents of β. The Ostrowski expansion of a real number x with respect to β is an expression of the form

$$x = \sum c_{n+1} D_n$$

subject to the condition that if $c_{n+1} = b_{n+1}$, then $c_n = 0$.

In the sets we investigate, there are restrictions on the digits c_n appearing in the Ostrowski expansions of real numbers x with respect to a fixed β. In the simplest case where $\beta = [0;b,b,\ldots]$ we find that our digitally restricted sets have dimension $-\log\alpha/\log\beta$, where α is a real number which is naturally related to the number of allowed Ostrowski digits. On the other hand, if the coefficients of β’s continued fraction are allowed to grow, then the digitally restricted sets have a dimension which depends on that growth. We also study “fractal percolation” in this setting, where the digital restrictions are determined by a random process.

These results have applications in metric Diophantine approximation.

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at 7 p.m. in Park 245 or via Zoom

https://brynmawr-edu.zoom.us/j/95807982212?pwd=aXBBMnFZMUUyWDQ1S1d3TGozc0t5Zz09

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